PURE MATH LAB · A-LEVEL PURE 3

Top 10 Mark-Loss Patterns in CAIE Pure 3

Ten exact mistakes Year 13 students lose marks on.
One takeaway, one visual, one fix — per page.

Written by Kru Peat (Kasidej J.) for parents of A-Level Pure 3 students.
Free guide · 13 pages · 15-min read

10 patterns at a glance

Read the takeaway. If it sounds familiar to a recent test, that pattern is the one to fix first. Each pattern has a one-page breakdown in this guide.

01 · INTEGRATION When in doubt, LOG beats X. $\int x\ln(x)\,dx$	**02 · INTEGRATION** If the bottom won't factor, it's arctan, not ln. $\int \frac{1}{x^2+9}\,dx$
03 · DIFFERENTIATION Differentiate y, multiply by dy/dx. Every time. $x^2+y^2=25$	**04 · TRIGONOMETRY** What sits in front of the angle decides which identity to use. $a\sin\theta + b\cos\theta$
05 · LOGS & EXP Bring the variable down first. Then divide. $3^x = 20$	**06 · INTEGRATION** When the original integral returns, name it and solve. $I = \int e^x \sin x\,dx$
07 · NUMERICAL METHODS Sketch first. Pick x1 near the root the question asks for. $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$	**08 · DIFFERENTIAL EQNS** Variables must SPLIT before you integrate. $\frac{dy}{dx} = \frac{xy}{x^2+1}$
09 · PARAMETRIC dy/dx = (dy/dt) divided by (dx/dt). Always divide. $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$	**10 · VECTORS** Perpendicular = dot product zero. Parallel = scalar multiple. $\mathbf{a}\cdot\mathbf{b} = 0$

Want to know which patterns your child specifically hits? Take our 20-minute free diagnostic at puremath-diagnostic.netlify.app. You will get a 3-page Parent Report mapped to the CAIE 9709 mark scheme.

PATTERN 01 · INTEGRATION

When in doubt, LOG beats X.

THE PROBLEM

$$\int x \ln(x)\, dx$$

X WRONG

$$u = x$$

✓ RIGHT

$$u = \ln(x)$$

WHY IT HAPPENS	THE RULE
Students grab the algebraic term out of habit. After differentiating, the second integral becomes harder, not easier.	**LIATE order: Log, Inverse trig, Algebraic, Trig, Exponential. The one that appears first becomes u.**

PATTERN 02 · INTEGRATION

If the bottom won't factor, it's arctan, not ln.

THE PROBLEM

$$\int \frac{1}{x^2+9}\,dx$$

✗ WRONG

$$\ln|x^2+9|+C$$

✓ RIGHT

$$\tfrac{1}{3}\arctan(x/3)+C$$

WHY IT HAPPENS	THE RULE
ln only works when the denominator factors over real numbers. If the quadratic has no real roots, the answer is arctan.	**If you see 1 over (x squared plus a constant), the answer is one over a, arctan of x over a, plus C.**

PATTERN 03 · DIFFERENTIATION

Differentiate y, multiply by dy/dx. Every time.

THE PROBLEM

$$x^2 + y^2 = 25, \quad \text{find } \frac{dy}{dx}$$

✗ WRONG

$$2x + 2y = 0$$

✓ RIGHT

$$2x + 2y\frac{dy}{dx} = 0$$

WHY IT HAPPENS

y is a function of x, so the chain rule kicks in every time you touch a y. Forgetting the dy/dx breaks the whole problem.

THE RULE

Say it out loud while writing: 'differentiate y, then multiply by dy/dx.' Solve for dy/dx at the end.

PATTERN 04 · TRIGONOMETRY

What sits in front of the angle decides which identity to use.

THE PROBLEM

$$3\cos\theta + 4\sin\theta = R\sin(\theta + \alpha)$$

✗ WRONG

$$\alpha = 53.13^\circ$$

✓ RIGHT

$$\alpha = 36.87^\circ \ (\tan\alpha = 3/4)$$

WHY IT HAPPENS

There are four R-form variants. Students memorise one and apply it everywhere. If b is negative or you used the wrong identity, alpha lands in the wrong quadrant.

THE RULE

For R sin of theta plus alpha: R cos alpha equals b, R sin alpha equals a. Check the sign before declaring the answer.

PATTERN 05 · LOGS & EXP

Bring the variable down first. Then divide.

THE PROBLEM

$$3^x = 20$$

✗ WRONG

$$x = \ln 20/3$$

✓ RIGHT

$$x = \frac{\ln 20}{\ln 3} \approx 2.727$$

WHY IT HAPPENS

Students take ln of both sides but then divide by 3 instead of by ln 3. The power rule didn't get applied properly.

THE RULE

Take natural log of both sides, then use the power rule to bring x out as a coefficient. Then divide by ln of the base.

PATTERN 06 · INTEGRATION

When the original integral returns, name it and solve.

THE PROBLEM

$$I = \int e^x \sin x \, dx$$

✗ WRONG

restart from scratch

✓ RIGHT

$$2I = e^x(\sin x - \cos x)$$

WHY IT HAPPENS	THE RULE
After two rounds of by-parts, the integral you started with appears on the right side. Students panic and restart. It is an algebra problem, not a calculus problem.	**If the original returns, call it I. Move it to the left. Solve algebraically. Divide by the coefficient.**

PATTERN 07 · NUMERICAL METHODS

Sketch first. Pick x1 near the root the question asks for.

THE PROBLEM

$$f(x) = x^3 + x - 3, \quad x_1 = ?$$

✗ WRONG

$$x_1 = -2 \quad \to \quad \text{diverges}$$

✓ RIGHT

$$x_1 = 1 \quad \to \quad x_2 = 1.250$$

WHY IT HAPPENS

Newton-Raphson is local. Different starting points converge to different roots, or diverge entirely. A random x1 loses easy marks.

THE RULE

Evaluate f at integer points or sketch the curve. Pick x1 close to the root the question specifies. Then iterate.

PATTERN 08 · DIFFERENTIAL EQNS

Variables must SPLIT before you integrate.

THE PROBLEM

$$\frac{dy}{dx} = \frac{xy}{x^2+1}, \qquad y(0) = 2$$

✗ WRONG

ntegrate without separatin

✓ RIGHT

$$\int \frac{dy}{y} = \int \frac{x\,dx}{x^2+1}$$

WHY IT HAPPENS

Separable form means dy/dx equals g(x) times h(y). If x and y are added on the right, you cannot split. Students integrate anyway and get nonsense that looks plausible.

THE RULE

Before integrating, check that the equation can be rearranged as dy over h(y) equals g(x) dx. If not, switch method.

PATTERN 09 · PARAMETRIC

dy/dx = (dy/dt) divided by (dx/dt). Always divide.

THE PROBLEM

$$x = 2t^2, \; y = 3t^3, \quad \text{find } \frac{dy}{dx}$$

✗ WRONG

$$\frac{dy}{dx} = 9t^2$$

✓ RIGHT

$$\frac{dy}{dx} = \frac{9t^2}{4t} = \frac{9t}{4}$$

WHY IT HAPPENS

When x and y are both functions of t, dy over dx equals dy over dt, all divided by dx over dt. Students remember the top half and forget to divide.

THE RULE

Write dy over dx equals (dy over dt) divided by (dx over dt) on the same line. Compute both. Then divide.

PATTERN 10 · VECTORS

Perpendicular = dot product zero. Parallel = scalar multiple.

THE PROBLEM

$$\mathbf{a} = (2, k, 3),\ \ \mathbf{b} = (1, -2, 1),\ \ \mathbf{a} \perp \mathbf{b}$$

✗ WRONG

$$\mathbf{a} = \lambda \mathbf{b}$$

✓ RIGHT

$$\mathbf{a} \cdot \mathbf{b} = 0 \Rightarrow k = \frac{5}{2}$$

WHY IT HAPPENS

Students compress two conditions into one rule. They use the dot product when they should use scalar multiples, or vice versa, and lose easy marks.

THE RULE

Read the keyword. Perpendicular means dot product equals zero. Parallel means one vector is a scalar multiple of the other.

NEXT STEP

Find out which patterns your child hits.

Reading patterns is useful. Knowing exactly which ones your child loses marks on, with worked examples from the actual diagnostic, is more useful.

Start the free diagnostic
puremath-diagnostic.netlify.app · 20 minutes

48h
Parent Report PDF
3 pages: estimated grade, top 3 weak areas, pathway scenarios.

45min
Free Zoom diagnostic
Walk through the report with Kru Peat. No commitment.

Free
Self-study pack
If now is not the right time, we send a study pack instead.

Kru Peat (Kasidej J.)
B.Eng. First Class Honors, M.Sc. Two years tutoring A-Level Pure 3 on top of ten years of applied engineering. Pure Math Lab is a small online classroom teaching only A-Level Pure 3.
Contact: kasidej.ju@gmail.com